Proceedings of 7th Annual International Conference
Busan, Korea, October 24-26, 2002.

RELIABILITY MODELLING WITH FUZZY COVARIATES

R. Guo
Department of Statistical Sciences
University of Cape Town
Rondebosch 7700, Cape Town, The Republic of South Africa
Email: rguo@maths.uct.ac.za

C. E. Love
School of Business Administration
Simon Fraser University
Burnaby, B.C. V5A 1S6, Canada
Email: love@sfu.ca

ABSTRACT

Uncertainty is an intrinsic feature of data containing the underlying information of reliability engineering realities. Randomness and fuzziness are two different type uncertainties although there is certain link between them. Cox's PH (Proportional Hazards) models and Lawless and Thiagarajah's CIF (Conditional Intensity Function) models addressed the random uncertainty in a very general format. As a matter of reflection, conditional monitoring technique-related models, say, proportional intensity (hazards) models and their extensions are one of the quick growth areas in reliability engineering research. However the fuzzy uncertainty in reliability engineering data modelling has not addressed yet. The ignorance of fuzziness is inevitably challenging the accuracy of modelling as well as the legality on the philosophical ground. Various forms of fuzzy covariate reliability modeling are to be explored. Furthermore, the intensity modelling with fuzzy covariate structure and the model implications into plant maintenance are explored.

1. INTRODUCTION

Western science has gradually formed an undeniable tradition: scientific laws on nature and society are supposed to be verifiable facts backed by reproducible experiments or observations. However, speeding waves of industrialization and commercialisation are gradually washing away the tradition and inevitably affect the reliability engineering community that should be the closest to the engineering reality.

Philosophically speaking, both randomness and fuzziness are parts of the world reality. Every event occurred in realistic engineering fields, no matter how complicated it might be, always enjoys its own self-existence and self-specification and therefore it is objective and certain. On the other hand, the true states of existing real world are never fully grasped by decision makers. What is available about the real world is the information collected. Therefore in order to correctly utilize data information in decision-making process, we have to understand the fundamental and intrinsic feature of information, *uncertainty*. Randomness and fuzziness are two different type uncertainties. Randomness, which is logically the break down of the law of causality because of the lack of some conditions under which the event occurrence is inevitable, is the traditionally well-received formality of uncertainty in terms of the usage of probability calculus by science and engineering. However, just as L. A. Zadeh (1988) pointed, "it has become increasingly clear that there are some important facets of uncertainty which do not lend themselves to analysis by classical probability-based methods". Fuzziness, which is logically the break down of the law of excluding the middle, is neither well known nor largely ignored in the community of reliability engineering and management.

To be clear, first we must point out that in real world fuzziness is sometimes inherent to the objects observed and particularly related to the evolution characteristics in real world environments. *Fuzziness is an objectively existing nature of the real world*. In industrial environments, factors (or covariates) related to system operation always change and evolve. The pattern of change is in general not sudden jump from one level to another but more possibly evolving from one stratum to another. In other words, between two different strata, there exist some middle layers, at each layer in-between; event possesses features in various degrees from both strata. On the hand, it quite often we face such a situation where the outcomes of a statistical experiments are not a real number but described in nature language, say, "the maintenance was carried on as planned", "the behaviour of the operator is unexpected", etc. Such a kind fuzziness exists objectively whenever information extracted as a reflection from the interaction between human brains and the operating machine systems. Therefore the existing certainty and fuzzy uncertainty are mutually coherent two facets of the objective (engineering) world.

However, once the engineering information is collected, how to extract the useful part and to what extend to summarize it for further decision-making will inevitably introduce another kind of fuzziness, *subjective* fuzziness, because information extraction depend heavily upon the experiences and knowledge of the decision makers. Even for some events with certainty, the information or image in human brains might be fuzzy in formality.

Furthermore, in engineering practices, a certain amount of factors and information may be neither random nor fuzzy, which can be called unclear information. Such purely subjective and unclear information might be manipulated in terms of subjective membership function or subjective probability.

Uncertain information can be therefore classified into two categories: strong form and weak form. Information with randomness and fuzziness, which is typically objective uncertainty, belongs to strong form of uncertainty while information with unclearness and vagueness, which is typically subjective uncertainty, belongs to weak form of uncertainty. The weakness of unclearness can be understood in two aspects: firstly whenever unclearness coexists with randomness and fuzziness it can be shaded or covered by the later; secondly when unclearness exists alone it can be represented by subjective membership function or subjective probability.

It is a commonly accepted principle in science that diversification of the world requires different methodologies. The two forms of uncertainty therefore need different mathematical treatments. It is true that the objectiveness of membership quantifications and usages is still a debatable issue due to the fact that currently membership functions in applications are largely subjectively specified, however, it is wrong to claim simply the equivalence between membership function in fuzzy mathematics and the prior distribution in Bayesian statistics. Although there exists a duality between membership function and probability mathematically it is in no way to regard them as the same research object. Duality does not mean equivalence. Only in the case of weak form uncertainty, it might be true that there is no difference in nature between subjective membership function and subjective probability.

2. FUZZY RELIABILITY CALCULUS

It should be emphasized here that the term "reliability" here is referred to as the concept that a physical system exercises its designated *capacity* of completing its engineering *specified functionality* within the *specified time* under its engineering *specified conditions*. Engineering speaking, reliability concept involves four aspects or four factors of a system. By examining the reliability of a system it is readily to accept the fact that randomness and fuzziness uncertainty coexist in every aspect of the system reliability. The root concept of reliability engineering, reliability, should be actually a hybrid of the two uncertainties. Although all the four factors are random and fuzzy in nature, the final measurement of the system reliability is still expected to appear in a conventional positive number between zero and one rather a fuzzy number. Such a choice will not create an unacceptable gap from the current exercises in the reliability engineering community.

Traditionally, the random characteristic in the engineering information is facilitated by classical probability calculus. Therefore logically the coexistence of fuzziness and randomness in reliability engineering information require a fuzzy probability calculus that combines fuzzy and random elements from fuzzy calculus and classical probability calculus. Such combination would help to lay down a cornerstone of new calculus to reflect the nature of the reliability dynamics of engineering systems.

The key concept to extend the classical probability calculus toward the fuzzy probability calculus is the indicator function of a random event $A \in \Im$, a σ-field of Ω.

$$\vartheta_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \notin A \end{cases}$$

One fundamental fact is that

$$\Pr(A) = \int_\Omega \vartheta_A(\omega) dP$$

the right hand is an abstract Lebegue integral.

Classical probability calculus requires random event A is a common subset, i.e., for all $\omega \in A$, such a belonging relation is definite: it either belongs to A or it does not, there is no middle ground. Therefore classical probability calculus is short of the capability to describe fuzzy random events. Zedeh (1965) defined fuzzy set in terms of the extension to indicator function of a normal subset into membership function of a subset into membership function of a fuzzy set A is mapping from Ω onto $[0,1]$

$$\mu_{\underset{\sim}{A}} : \Omega \to [0,1].$$

This mapping is called the membership function of $\underset{\sim}{A}$, which is a Borel measurable function representing the degree of element ω belonging to fuzzy set. Thus the probability of fuzzy event is defined as

$$\Pr\left[\underset{\sim}{A}\right] = \int_\Omega \mu_{\underset{\sim}{A}}(\omega) dP.$$

Given a probability space $(\Omega, \Im, P)$, let $\wp$ be the collection of all the fuzzy event on Ω, then $(\Omega, \wp, P)$ is called the induced fuzzy probability space from $(\Omega, \Im, P)$. Therefore, the fuzzy probability calculus can be established naturally as the extension to the classical probability calculus except the membership of the interception of two fuzzy events

$$\mu_{\underset{\sim}{A} \cap \underset{\sim}{B}} \triangleq \mu_{\underset{\sim}{A}} \cdot \mu_{\underset{\sim}{B}}$$

for maintaining the classical formality of independence, conditional probability, law of total probability as well as Bayes formula.

3. COVARIATE MODELLING

Based on the foundation of the fuzzy probability calculus, reliability modelling incorporating the covariate information associated with system operation may be developed in various manners.

3.1 An additive covariate model

Starting from the definition of fuzzy reliability

$$R^s(t) = \int_{-\infty}^{\infty} f(u,t) \mu_{\underset{\sim}{\Omega}}(u) du.$$

The fuzzy probability is the product of the two terms.

$$R^s(t) = \varepsilon(t) R(t),$$

where

$$\varepsilon(t) = \frac{\int_{-\infty}^{\infty} f(u,t)\mu_{\underset{\sim}{A}}(u)du}{\int_{-\infty}^{\infty} f(u,t)\vartheta_J(u)du}$$

Differentiating the negative log of both sides gives the fuzzy hazard rate function

$$\begin{aligned} h^{\varepsilon}(t) &= \frac{d}{dt}[-\ln R_{\varepsilon}(t)] = \frac{d}{dt}[-\ln(\varepsilon(t)R(t))] \\ &= h(t) + \frac{d}{dt}[-\ln\varepsilon(t)] = h(t) + g\left(\underset{\sim}{z}(t)\right) \end{aligned}$$

It is quite obvious that the additive fuzzy covariate model is a natural result based on the fuzzy reliability definition. The positive real-valued function may takes the form like $g(\beta'\underline{z}(t))$.

3.2 The multiplicative fuzzy covariate models

Another way to incorporate the fuzzy covariate information to the reliability modelling is to examine the fuzzy hazard function in terms of the fuzzy reliability.

$$R^{\varepsilon}(t) = e^{-\int_0^t h_{\varepsilon}(u)du};$$

The fuzzy reliability model should fully contain the random and fuzzy inputs from the system operating conditions, i.e., the "specified conditions", they can be roughly classified into four categories: the environmental conditions, the applying conditions, the operating conditions and the maintaining conditions of a system. This is an extension of Cox's proportional hazard model

$$h_{\varepsilon}(t) = g(\underset{\sim}{z})h_0(t)$$

where $h_0(t)$ is the baseline hazard function which reflects the risk of the system under the standard working conditions, and vector z represents the working variables coexisting with system's functioning. A general model is an extension to the model which originally proposed by Ciampi and Etezadi-Amoli (1985) and further explored by Love and Guo (1995), which takes a form

$$h_{\varepsilon}(t) = \xi(\underset{\sim}{z})h_0(\psi(\underset{\sim}{z})t).$$

An extension to Lawless and Thiagarajah's CIF (Conditional Intensity Function) (1996) model is

$$\lambda(t|\mathcal{A}_t) = e^{\theta^T \underset{\sim}{z}(t)}$$

3.3 Fuzzy covariates with triangle membership functions

The simplest membership function is of a triangle form.

The sum of a linear combination of fuzzy numbers with triangle membership functions also possesses a triangle membership function. In terms of such a simplification, the fuzzy hazard of a system can be shown that it is of multiplicative form if the baseline hazard takes power law form.

4. ESTIMATION METHODS

The system operating dynamics is in nature a general counting process. Although it needs less assumptions, but its two fundamental properties which lead to applications greatly simplified. First, we notice that a counting process $\{N_t\}$ can be decomposed into two parts: a system term, represented by the compensator $\{A_t\}$, a smoothly varying and predictable process, and a pure "noise" term represented by a martingale $\{M_t\}$ with a unpredictable zero-mean

$$E(dM_t|F_{t-})=0,$$

i.e.,

$$M_t=N_t - A_t.$$

Secondly, we notice a local Poisson character of a counting process. In other words, under certain conditions (particularly, the continuity and predictability of the compensator A_t are assumed) the conditional mean and the conditional variance of the increment are equal, i.e.,

$$E(dN_t|\mathcal{F}_{t-}) = \mathrm{VAR}(dN_t|\mathcal{F}_{t-}) = dA_t$$

These remarkable simple facts will not only help us to understand the basic feature of the point counting processes but also provide some clues to estimate the parameters of counting point processes.

The stochastic intensity takes typically the form as

$$\lambda_{\varepsilon}(t) = Y(t)h_{\varepsilon}(t)$$

where

$$Y(t) = \#\left\{i : \tilde{T}_i \geq t\right\}.$$

Then the semi-parametric or non-parametric estimation techniques for counting point processes with covariates could be extended into fuzzy covariate cases. Particularly, the Cox's partial likelihood estimation is useful here.

$$C_t(\boldsymbol{\beta}) = \sum_{h=1}^{k}\left[\sum_{i=1}^{n}\int_0^T \ln(\boldsymbol{\beta}^T \underset{\sim}{\mathbf{z}_{hi}}(t))dN_{hi}(t) - \int_0^T \ln S_h^{(0)}(\boldsymbol{\beta},t)dN_h(t)\right]$$

5. OPTIMAL MAINTENANCE

In the handbook of reliability engineering edited by I. A. Ushakov (1994), four indices in maintenance analysis are proposed: availability coefficient, the operational availability coefficient, and average profit from the system unit time and the average total cost per unit time. The optimisations are under a PM or a repair is restoring the system into same-as-new state. Love and Guo (1991) investigated same-as-old (bad-as-old) and same-as-new (good-as-new) PM cycle times involving covariates, and also by Ascher and Kobbacy (1997). Unfortunately, as Guo and Love (1995) pointed out both same-as-new and same-as-old assumptions is unrealistic approximation to the real engineering realities. Kumar and Westberg studied the maintenance problem under age model as well as PH covariates. However, the fundamental issue here is that the imposing many restrictions on reliability modelling and maintenance optimisation is not a final solution to engineering community. Therefore, we propose studying it under general counting processes framework, say, Last and Brandt (1995), Baxter, Kijima and Tortorella (1996).

Assume a general counting point process as underlying process. The optimal criteria used are to find the PM cycle length that minimizes total cost per time unit. For $t \geq 0$, let $N(t,z)$ be the number of failures in time period $(0,t)$ for a counting process (including same-as-old (Poisson) process as its special case) with fuzzy covariate vector z , let C_r be the average emergency repair cost and C_{PM} be the average PM cost, then the maintenance cost per unit time can be obtained by minimizing the unit time cost function

$$C(t, \underset{\sim}{z}) = \frac{1}{t}\left[C_r E\left(N(t, \underset{\sim}{z})\right) + C_{PM}\right]$$

Under fuzzy probability calculus framework, the repair costs and the PM costs initially are not necessarily exact figures and they are possible fuzzy figures. The expected occurrences

$$\begin{aligned} E\left(N(t, \underset{\sim}{z})\right) &= A(t, \underset{\sim}{z}) \\ &= \int_0^t \lambda_z(u)\,du \\ &= \int_0^t Y(u) h_0(u) g(\beta^T \underset{\sim}{z})\,du \end{aligned}$$

Also, we need to emphasize that the time t in the formula is actual the fuzzy equivalent time (not the calendar time at all). The final computation may involve a defuzzification technique. Other optimal criteria could be similarly developed with care.

6. CONCLUDING REMARKS

In this paper, we argued philosophically and logically that fuzziness is an intrinsic element of uncertainty underlying the operating system information. Therefore reliability modelling based on either fuzziness or randomness alone is not the solution but we ought to combine fuzziness and randomness together in reliability modelling. The remaining arguments are intended for facilitating a frame of statistical inference on system reliability that could incorporate both random and fuzzy information. This frame is based on the fuzzy reliability formulation and an extension of the general counting point process associated with covariate information. Some aspects of fuzzy reliability calculus, general counting processes with fuzzy covariates, statistical estimation and optimal maintenance are discussed. However, we have to admit that much more refinements need to be done for theoretical soundness and promotion to the real industrial applications.

7. REFERENCES

Andersen, P.K., Borgan, Ø., Gill, R.D. and Keiding, N. Statistical Models Based on Counting Processes, Springer-Verlag, 1993

Ascher, H. E. and Kobbacy, K.A.H. Modelling Preventive Maintenance for Deterirating Repairable System. IMA Journal of Mathematics Applied in Business & Industry 6, 85-99, Oxford University Press, 1995

Baxter, L.A., Kijima, M., and Tortorella, M. (1996) A Point Process Model for the Reliability of a Maintained System Subject to General Repair. Communications in Statistics-Stochatistic Models 1, 37-65, 1996

Cinlar, E. and Ozekici, S. Reliability of Complex Devices in Random Environment. Probability in the Engineering and Information Sciences 1, 97-115, 1989

Ciampi, A. and Etezadi-Amoli, J. A General Model for Testing the Proportional Hazards and the Accelerated Failure Time Hypothesis in the Analysis of Censored Survival Data with Covariates. Communication in Statistics-Theory and Methods 14(3) 651-667, 1985

Guo, R. and Love, C.E. Bad-As-Old Modelling of Complex Systems with Imperfectly Repaired Subsystems. Proceedings of the International Conference on Statistical Methods and Statistical Computing for Quality and Productivity Improvement, August 17-19, Seoul, Korea, 131-140, 1995

Kumar, D. and Westberg, U. Maintenance Scheduling under Age Replacement Policy Using Proportional Hazards Model and TTT-plotting. European Journal of Operational Research 99, 493-506, 1997

Last, G. and Brandt, A. Marked Point Processes on the Real Line: the Dynamic Approach, Springer, 1995

Lawless, J.F. Statistical Models and Methods for Lifetime Data. Wiley, New York (1983)

Lawless, J.E. and Thiagarajah, K. A Point-Process Model Incorporating Renewal and Time Trend, with Application to Repairable Systems. Technometrics 38, 131-138, 1996

Love, C.E. and Guo, R. Using Proportional Hazard Modelling in Plant Maintenance. Quality and Reliability Engineering International Vol. 7, 7-17, 1991

Love, C.E. and Guo, R. Statistical Analysis of General Covariate Structure. Proceedings of the International Conference on Statistical Methods and Statistical Computing for Quality and Productivity Improvement, August 17-19, Seoul, Korea, 131-140, 1995

Love, C.E., Zhang, Z.G., Zitron, M.A. and Guo, R. A discrete Semi-Markov Decision Model to Determine the Optimal Repair/Replacement Policy under General Repairs. European Journal of Operational Research 125, 398-409, 2000

Ushakov I. A. and Harrison, R. A. Handbook of Reliability Engineering. John Willey & Sons, Inc. New York, 1994

Zadeh, A. Fuzzy sets, Information and Control 8, 338-353, 1965

Zadeh, A. *Forward* of " Possibility Theory" by Dubois, D. and Prade, H. Plenum Press, New York and London, 1988